# Douglas Cluster

## Numeracy

## A Guide

# Introduction

**What is Numeracy?**

*Numeracy is a skill for life, learning and work. Having well-developed numeracy skills allows young people to be more confident in social settings and enhances enjoyment in a large number of leisure activities.*

Numeracy across Learning: Principles & Practice

**What is the purpose of the booklet?**

This booklet has been produced to give guidance to staff & parents/carers on how certain common numeracy topics are taught within the Mathematics department for problem solving, following the Curriculum for Excellence guidelines used in all schools in Scotland.

<u>Curriculum for Excellence Numeracy Strands</u>

- Number and number processes

**How can it be used?**

Before teaching a topic containing numeracy you can refer to the booklet to see what methods are being taught. If your daughter or son is working on numeracy at home then you may wish to use this booklet to see what methods are used in school so you can assist them. A timeline of when topics are taught in S1/S2 is also included in this booklet.

**Why do some topics include more than one method?**

For mental calculations, pupils should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.

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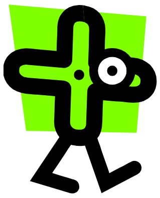
# Addition

## Mental strategies



Children will be taught a variety of mental addition strategies and are encouraged to select the most appropriate method in any given situation. Some examples are given below.

**Example** Calculate 64 + 27

**Method 1** **Partitioning**

Add tens, then add units, then add together

60 + 20 = 80 4 + 7 = 11 80 + 11 = 91

**Method 2** **Fail Safe**

Split up number to be added (last number 27) into tens and units and add separately.

64 + 20 = 84 84 + 7 = 91

**Method 3** **Rounding & Adjusting**

Round up to nearest 10, then subtract

64 + 30 = 94 but 30 is 3 too much so subtract 3;
94 - 3 = 91

**Method 4** **Transforming**

Take from one number to bring the other number to a multiple of 10.

3

64 + 27

Take 3 from 64 and add to 27 to create the easier calculation of 61 + 30 = 91

OR

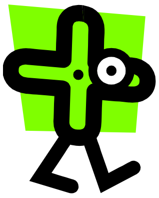
Take 6 from 27 and add to 64 to create the easier calculation of 70 + 21 = 91

6

64 + 27

# Addition

## Mental strategies



**Method 5** **Friendly Pairs**

When adding multiple numbers, look for number bonds to 10.

63 + 12 + 27 Adding the 3 + 7 allows the mental calculation to be simplified to 63 + 7 + 12 + 20

= 70 + 12 + 20

= 82 + 20

## Written Method

*This is also known as the formal method, vertical addition, a chimney sum, an up and down sum.*

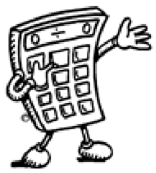
When adding numbers, careful layout of working is essential. Ensure that the numbers are lined up according to place value. Start at right hand side, write down units digit, carry tens digit.

**Example** Add 3032 and 589

| | | | |
|---|---|---|---|
| Th H T U | Th H T U | Th H T U | Th H T U |
| 3 0 3 2 | 3 0 3 2 | 3 0 3 2 | 3 0 3 2 |
| + 5 8 9 | + 5 8 9 | + 5 8 9 | + 5 8 9 |
| $^{1}1$ | $^{1}2^{1}1$ | $6^{1}2^{1}1$ | $3\ 6^{1}2^{1}1$ |
| 2 + 9 = 11 | 3+8+1=12 | 0+5+1=6 | 3 + 0 = 3 |

N.B. Carried digits may be placed either above or below the answer line.

# Subtraction



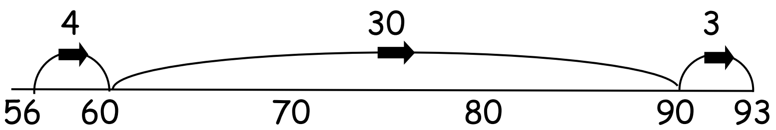
> We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

## Mental Strategies

**Example** Calculate 93 - 56

**Method 1** **Count On** (to find the difference between the numbers)

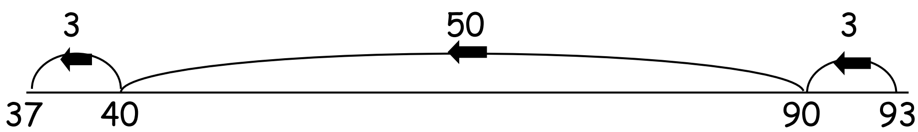
Count on from 56 until you reach 93. This can be done in several ways,
e.g.



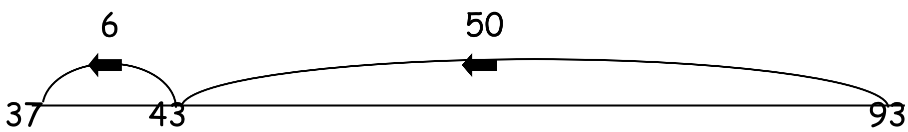

= 37

**Method 2** **Break It Down** (break up the number being subtracted)

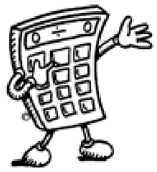
e.g. subtract 3, subtract 50, then subtract 3




OR subtract 50 then subtract 6



# Subtraction



## Mental Strategies

**Example** Calculate 93 - 56

**Method 3** **Rounding and Adjusting**
Round up to nearest 10, then subtract

93 - 60 = 33 but 60 is 4 too much so add 4 back on;
33 + 4 = 37

## Written Method

When subtracting numbers, careful layout of working is essential. Ensure that the numbers are lined up according to place value. Start at the right-hand side.

**Example 1** 4590 – 386 **Example 2** Subtract 692 from 14597

Important steps for **exchanging** (example 1)

1. Say "zero subract 6, we can't do this"
2. Look to next column and **exchange** one ten for ten units, i.e. 9 tens becomes 8 tens & 10 units
3. Then say "ten take away six equals four"
4. Normal subtraction rules can then be used to complete the calculation.

$$\begin{array}{r} 45\overset{8}{\cancel{9}}\,{}^{1}0 \\ -\ 386 \\ \hline 4204 \\ \hline \end{array} \qquad \begin{array}{r} 1\overset{3}{\cancel{4}}\,{}^{1}597 \\ -\ \ 692 \\ \hline 13905 \\ \hline \end{array}$$

# Multiplication 1

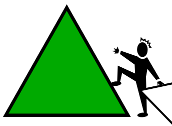
It is essential that you know all of the multiplication tables from 1 to 10 with accurate and fast recall. These are shown below.

| X1 | X2 | X3 | X4 | X5 |
|---|---|---|---|---|
| 1 x 0 = 0 | 2 x 0 = 0 | 3 x 0 = 0 | 4 x 0 = 0 | 5 x 0 = 0 |
| 1 x 1 = 1 | 2 x 1 = 2 | 3 x 1 = 3 | 4 x 1 = 4 | 5 x 1 = 5 |
| 1 x 2 = 2 | 2 x 2 = 4 | 3 x 2 = 6 | 4 x 2 = 8 | 5 x 2 = 10 |
| 1 x 3 = 3 | 2 x 3 = 6 | 3 x 3 = 9 | 4 x 3 = 12 | 5 x 3 = 15 |
| 1 x 4 = 4 | 2 x 4 = 8 | 3 x 4 = 12 | 4 x 4 = 16 | 5 x 4 = 20 |
| 1 x 5 = 5 | 2 x 5 = 10 | 3 x 5 = 15 | 4 x 5 = 20 | 5 x 5 = 25 |
| 1 x 6 = 6 | 2 x 6 = 12 | 3 x 6 = 18 | 4 x 6 = 24 | 5 x 6 = 30 |
| 1 x 7 = 7 | 2 x 7 = 14 | 3 x 7 = 21 | 4 x 7 = 28 | 5 x 7 = 35 |
| 1 x 8 = 8 | 2 x 8 = 16 | 3 x 8 = 24 | 4 x 8 = 32 | 5 x 8 = 40 |
| 1 x 9 = 9 | 2 x 9 = 18 | 3 x 9 = 27 | 4 x 9 = 36 | 5 x 9 = 45 |
| 1 x 10 = 10 | 2 x 10 = 20 | 3 x 10 = 30 | 4 x 10 = 40 | 5 x 10 = 50 |

| X6 | X7 | X8 | X9 | X10 |
|---|---|---|---|---|
| 6 x 0 = 0 | 7 x 0 = 0 | 8 x 0 = 0 | 9 x 0 = 0 | 10 x 0 = 0 |
| 6 x 1 = 6 | 7 x 1 = 7 | 8 x 1 = 8 | 9 x 1 = 9 | 10 x 1 = 10 |
| 6 x 2 = 12 | 7 x 2 = 14 | 8 x 2 = 16 | 9 x 2 = 18 | 10 x 2 = 20 |
| 6 x 3 = 18 | 7 x 3 = 21 | 8 x 3 = 24 | 9 x 3 = 27 | 10 x 3 = 30 |
| 6 x 4 = 24 | 7 x 4 = 28 | 8 x 4 = 32 | 9 x 4 = 36 | 10 x 4 = 40 |
| 6 x 5 = 30 | 7 x 5 = 35 | 8 x 5 = 40 | 9 x 5 = 45 | 10 x 5 = 50 |
| 6 x 6 = 36 | 7 x 6 = 42 | 8 x 6 = 48 | 9 x 6 = 54 | 10 x 6 = 60 |
| 6 x 7 = 42 | 7 x 7 = 49 | 8 x 7 = 56 | 9 x 7 = 63 | 10 x 7 = 70 |
| 6 x 8 = 48 | 7 x 8 = 56 | 8 x 8 = 64 | 9 x 8 = 72 | 10 x 8 = 80 |
| 6 x 9 = 54 | 7 x 9 = 63 | 8 x 9 = 72 | 9 x 9 = 81 | 10 x 9 = 90 |
| 6 x 10 = 60 | 7 x 10 = 70 | 8 x 10 = 80 | 9 x 10 = 90 | 10 x 10 =100 |

# Multiplication 2

## Multiplying by multiples of 10 and 100



To multiply by **10** you move every digit ***one*** place to the left and, if necessary, insert a zero to the units column as a place holder.
To multiply by **100** you move every digit ***two*** places to the left and, if necessary, insert two zeros to the tens and units columns as place holders.

**Example 1** (a) Multiply 354 by 10   (b) Multiply 50.6 by 100

| Th | H | T | U |
|---|---|---|---|
| | 3 | 5 | 4 |
| 3 | 5 | 4 | 0 |

354 x 10 = 3540

| Th | H | T | U | • | t |
|---|---|---|---|---|---|
| | | 5 | 0 | • | 6 |
| 5 | 0 | 6 | 0 | • | 0 |

50.6 x 100 = 5060

(c) 6 x 30

To multiply by 30, multiply by 3, then by 10.

6 x 3 = 18
18 x 10 = 180
so 6 x 30 = 180

## Mental Strategies

**Example 1** Find 39 x 6

**Method 1 Split Method**

30 x 6
= 180

\+

9 x 6
= 54

=

180 + 54
= 234

**Method 2 Rounding & Adjusting**

40 x 6
=240

40 is 1 too many
so take away 1x6

240 - 6
= 234

# Multiplication 3

**Example** Find 15 x 12

**Method 3** **Doubling and Halving**

When mentally multiplying double-digit numbers (if one number is even and the other is odd) it can make the calculation easier if you double the odd number and half the even number before performing the calculation.

15 x 2 = 30 × 12 ÷ 2 = 6 = 30 x 6 = 180

# Multiplication 4

## Written Method

When multiplying numbers, careful layout of working is essential.
Start at the right-hand side.

**Example 1** Multiply 28 by 3.

$$\begin{array}{r} 28 \\ \times \quad 3 \\ \hline 4 \\ \hline {\scriptstyle 2} \end{array} \qquad\qquad \begin{array}{r} 28 \\ \times \quad 3 \\ \hline 84 \\ \hline {\scriptstyle 2} \end{array}$$

3 x 8 = 24
Write units digit
(4), carry tens
digit (2).

3 x 2 = 6
Remember to add on
the carried digit.
6 + 2 = 8

n.b. The carried digit can be recorded above or below the answer line.

**Example 2** Multiply 28 by 30.

$$\begin{array}{r} 28 \\ \times \; 30 \\ \hline 0 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 28 \\ \times \quad 30 \\ \hline 40 \\ \hline {\scriptstyle 2} \end{array} \qquad\qquad \begin{array}{r} 28 \\ \times \quad 30 \\ \hline 840 \\ \hline {\scriptstyle 2} \end{array}$$

28 x 3 x 10

To ensure we multiply by 30, insert a zero in the units as a place holder. Now multiply by 3 as in Example 1.

# Multiplication 5

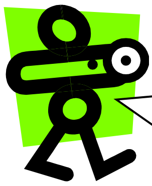
**Written Method -** Long multiplication

**Example** **Find 356 x 48**

| | | |
|---|---|---|
| Step 1:- | multiply the 356 by the 8 (= 2848). | |
| Step 2:- | now multiply by 40, <u>not 4</u> (= 14240). (it's easier to place a zero below the 2 then multiply by the 4). | |
| Step 3:- | now simply add together your answers. | |

```
   3 5 6
  x  4 8
  -------
  2 8 4 8
1 4 2 4 0
---------
1 7 0 8 8
---------
```

# Division



You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.
A sound working knowledge of multiplication tables is essential for division.

## Mental Strategies

**Method 1** Simple mental division

$20 \div 4$ can also be written as $\frac{1}{4}$ of 20.

4 x ? = 20

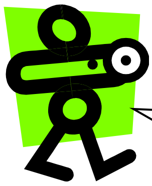
$20 \div 4 = ?$

4 x 5 = 20

$20 \div 4 = 5$

Use your knowledge of times tables facts.

**Method 2** Dividing by multiplies of 10, 100



To divide by **10** you move every digit ***one*** place to the right.
To divide by **100** you move every digit ***two*** places to the right.

**Example** (a) Divide 3540 by 10 (b) Divide 5060 by 100

| Th | H | T | U |
|---|---|---|---|
| 3 | 5 | 4 | 0 |
| | 3 | 5 | 4 |

$3540 \div 10 = 354$

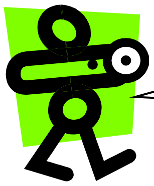
| Th | H | T | U | • | t |
|---|---|---|---|---|---|
| 5 | 0 | 6 | 0 | • | 0 |
| | | 5 | 0 | • | 6 |

$5060 \div 100 = 50.6$

(c) $180 \div 30$
$180 \div 10 = 18$, $18 \div 3 = 6$
$180 \div 30 = 6$

Divide by 10
then divide by 3

# Division



Not all divisions work out exactly! This can be recorded with remainders.

**Division with Remainders**

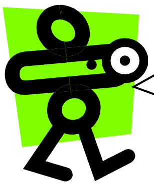
**Example** $50 \div 7$

$7 \times 7 = 49$

$50 - 49 = 1$

$50 \div 7 = 7 \text{ r } 1$

# Division



You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.
A sound working knowledge of multiplication tables is essential for division.

**Written Method**

**Example 1** 192 ÷ 8

**Important steps for dividing (Example 1)**

1. Divide from left to right.
2. 8 goes into 1 zero times. Record the zero and carry the remainder to the next column.
3. 8 goes into 19 2 times with a remainder of 3. Record the 2 carry the 3 to the next column.
4. 8 goes into 32 4 times. Record the 4.

$$\begin{array}{r} 2\ 4 \\ 8\overline{)\,1^{1}9^{3}2} \end{array}$$

**Example 2** Divide 4.74 by 3

$$\begin{array}{r} 1\ .\ 5\ 8 \\ 3\overline{)\,4\ .\ ^{1}7^{2}4} \end{array}$$

**When dividing a decimal number by a whole number, the decimal points must stay in line.**

**Example 3** 2.2 ÷ 8

$$\begin{array}{r} 0\ .\ 2\ 7\ 5 \\ 8\overline{)\,2\ .\ ^{2}2^{6}0^{4}0} \end{array}$$

**If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.**

**Long Division** – This is no longer part of the curriculum. Pupils would estimate the answer and then use a calculator to get the exact answer.

# Order of Calculation (BODMAS)

Consider this: What is the answer to 2 + 4 x 5 ?

Is it (2+4) x 5 or 2 + (4 x 5)
= 6 x 5 = 2 + 20
= 30 = 22

The correct answer is 22.

Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BODMAS.**

***n.b. Division and multiplication have equal priority, as do addition and subtraction. Always work from left to right.***

The **BODMAS** rule tells us which operations should be done first.

**BODMAS** represents:

**(B)rackets**
**(O)rder (or 'OF' as in 'power of')**
**(D)ivide**
**(M)ultiply**
**(A)dd**
**(S)ubract**

Therefore in the example above multiplication should be done before addition. (Note order means a number raised to a power such as $2^2$ or $(-3)^3$)

Scientific calculators are programmed with these rules, however some basic calculators may not, so take care.

**Example 1** 15 – 12 ÷ 6 BODMAS tells us to divide first
= 15 – 2
= 13

**Example 2** (9 + 5) x 6 BODMAS tells us to work out the brackets first
= 14 x 6
= 84

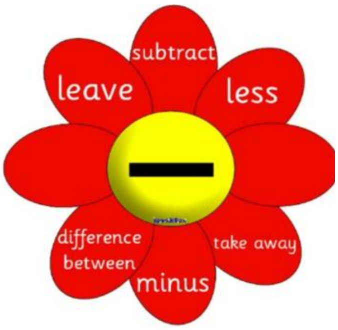
**Example 3** 18 + 6 ÷ (5-2) Brackets first
= 18 + 6 ÷ 3 Then divide
= 18 + 2 Now add
= 20

**Example 4** 18 - 6 + 2 Add & subtract have equal priority
= 12 + 2 So subtract
= 14 Then add

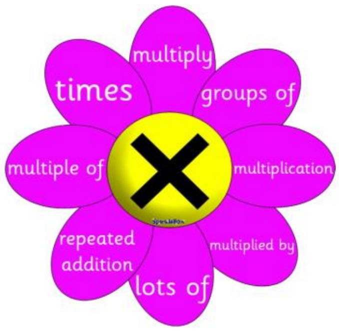
# Operational Mathematical Literacy

***Word problem tips:***
***Look for any key words. Do you expect a bigger or smaller answer? Use this to help you decide which operation to use.***

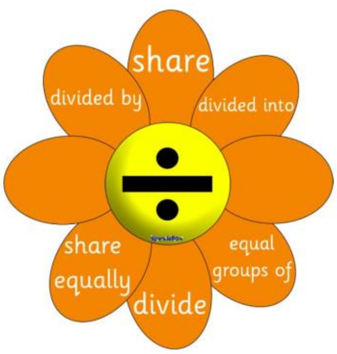

By the end of 2016 my car had travelled 8550 miles. During 2017 I drove a further 6400 miles. How far had my car travelled by the end of 2017?




During a storm a plane dropped from 34 5000 feet to 30 200 feet. By how much had it dropped?

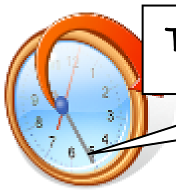



A factory packages cans of soup in boxes. There are 24 cans in each box. What is the product of 8 boxes?




There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

# Time 1



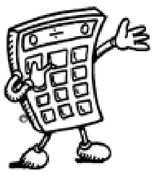
Time may be expressed in 12 or 24 hour notation.

**12-hour clock** Time can be displayed on a clock face, or digital clock.

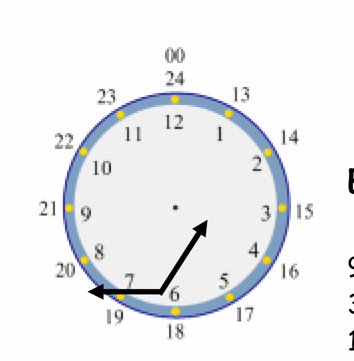

These clocks both show fifteen minutes past five, or quarter past five.

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.
a.m. is used for times between midnight and 12 noon (morning).
p.m. is used for times between 12 noon and midnight (afternoon / evening).

## 24-hour clock



In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 00 00, or 24 00. After 12 noon, the hours are numbered 13, 14, 15 … etc.




**Examples**

9.55 am → 09 55 hours
3.35 pm → 15 35 hours
12.20 am → 00 20 hours
02 16 hours → 2.16 am
20 45 hours → 8.45 pm

Reading timetables

When reading timetables you often have to convert to and from 24 hours clock.

**To convert from 24 hour time to 12 hour time:**

A. If the hour is 13 or more, subtract 12 from the hours and call it P.M. Otherwise it is A.M.
B. If the hour is 12, leave it unchanged, but call it P.M.
C. If the hour is 0, make it 12 and call it A.M.
D. Otherwise, leave the hour unchanged and call it A.M.

**To convert from 12-hour time to 24-hour time:**

A. If the P.M. hour is from 1 through 11, add 12.
B. If the P.M. hour is 12, leave it as is.
C. If the A.M. hour is 12, make it 0.
D. Otherwise, leave the hour unchanged.
Then drop the A.M. or P.M., of course.

***************Check rules with examples above***********************

# Time 2

## Time Facts

It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

## Time Calculations

**Example 1** How long is it from 0755 to 0948?

Method - Working ( use a time line)

0755 -> 0800 -> 9000 -> 0948

(5mins) + (1hr) + (48mins)

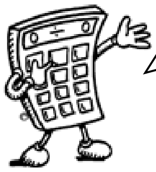
***WE DON'T TEACH TIME AS SUBTRACTION***

**Example 2** Change 27 minutes into hours equivalent

27mins = 27÷60

= 0.45 hours

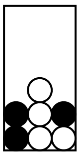
# Fractions 1



Addition, subtraction, multiplication and division of fractions are studied in mathematics.
However, the examples below may be helpful in all subjects.

## Understanding Fractions

**Example**
A jar contains black and white sweets.

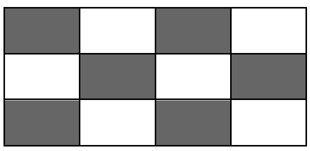


What fraction of the sweets are black?

There are 3 black sweets out of a total of 7, so $\frac{3}{7}$ of the sweets are black.

## Equivalent Fractions

**Example**
What fraction of the flag is shaded?



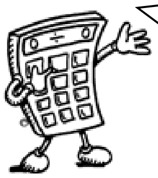
6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

# Fractions 2

## Simplifying Fractions



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.
To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same number.

**Example 1**

(a) $\frac{20}{25} \underset{\div 5}{\overset{\div 5}{=}} \frac{4}{5}$

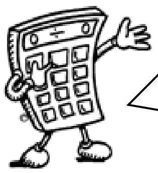
(b) $\frac{16}{24} \underset{\div 8}{\overset{\div 8}{=}} \frac{2}{3}$

This can be done repeatedly until the numerator & denominator are the smallest possible numbers - the fraction is in it's **simplest form**.

**Example 2** Simplify $\frac{72}{84}$

$\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (simplest form)

## Calculating Fractions of a Quantity



To find the fraction of a quantity, divide by the denominator.
To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

**Example 1**

Find $\frac{1}{5}$ of £150

$\frac{1}{5}$ of £150 = £150 ÷ 5 = £30

**Example 2** Find $\frac{3}{4}$ of 48

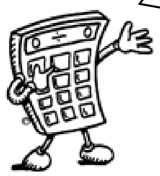
$\frac{1}{4}$ of 48 = 48 ÷ 4 = 12

so $\frac{3}{4}$ of 48 = 3 x 12 = 36

To find $\frac{3}{4}$ of a quantity, start by finding $\frac{1}{4}$ then multiply by 3 (the numerator)

# Fractions 3

## Adding, Subtracting Fractions



To add and subtract fractions you have to make the denominators the same by using equivalent fractions. Then you add or subtract the new numerators.

**Example 1**

$$\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$$

Both fractions have the same denominator so we can simply add the numerators

**Example 2**

$$\frac{2}{3}-\frac{1}{4}$$

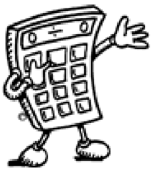
$$=\frac{8}{12}-\frac{3}{12}$$

$$=\frac{5}{12}$$

Make denominators the same. Then subtract new numerators.

# Fractions 4

## Multiplying, Dividing Fractions



To multiply fractions multiply the numerators together and the denominators together separately. To divide turn the second fraction upside down (flip) and then multiply.

**Example 1**

$$\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$$

**Example 2**

$$\frac{2}{7} \div \frac{1}{3}$$

$$= \frac{2}{7} \times \frac{3}{1}$$

$$= \frac{6}{7}$$

Remember to flip the second fraction!

**Example 3**

$$\frac{4}{5} \times \frac{35}{36}$$

Divide 4 into 36 and 5 into 35 to get:

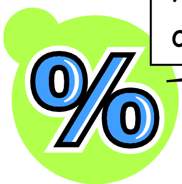
$$\frac{1}{1} \times \frac{7}{9}$$

$$= \frac{7}{9}$$

In cases of larger numbers it is easier to simplify these down by finding a number that divides into the numerator on the left hand-side and denominator on the right hand side fraction, then finding a number that divides the denominator on the left hand-side and the numerator on the right.

# Percentages 1

Percent means out of 100.
A percentage can be converted to an equivalent fraction or decimal.



36% means $\frac{36}{100}$

36% is therefore equivalent to $\frac{9}{25}$ and 0.36

*To change a fraction to a decimal (fraction) divide the numerator by the denominator*

## Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals.

| Percentage | Fraction | Decimal (Fraction) |
|---|---|---|
| 1% | $\frac{1}{100}$ | 0.01 |
| 10% | $\frac{1}{10}$ | 0.1 |
| 20% | $\frac{1}{5}$ | 0.2 |
| 25% | $\frac{1}{4}$ | 0.25 |
| $33\frac{1}{3}\%$ | $\frac{1}{3}$ | 0.333… |
| 50% | $\frac{1}{2}$ | 0.5 |
| $66\frac{2}{3}\%$ | $\frac{2}{3}$ | 0.666… |
| 75% | $\frac{3}{4}$ | 0.75 |

# Percentages 2

There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

## Non- Calculator Methods

**Method 1 Using Equivalent Fractions**

**Example** Find 25% of £640

25% of £640 $= \frac{1}{4}$ of £640 = £640 ÷ 4 = £160

**Method 2 Using 1%**

First find 1% of the quantity (by dividing by 100), then multiply to give the required value.

**Example** Find 9% of 200g

1% of 200g = $\frac{1}{100}$ of 200g = 200g ÷ 100 = 2g

so 9% of 200g = 9 x 2g = 18g

**Method 3 Using 10%**

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

**Example** Find 70% of £35

10% of £35 = $\frac{1}{10}$ of £35 = £35 ÷ 10 = £3.50

so 70% of £35 = 7 x £3.50 = £24.50

Or find 30% and then subtract this.

10% of £35 = $\frac{1}{10}$ of £35 = £35 ÷ 10 = £3.50

So 30% of £35 = 3 x £3.50 = £10.50

100% - 30% = £35.00 – £10.50 = £24.50

# Percentages 3

## Non- Calculator Methods (continued)

The previous methods can be combined so as to calculate any percentage.

**Example** Find 23% of £15000

10% of £15000 = £1500 so 20% = £1500 x 2 = £3000
1% of £15000 = £150 so 3% = £150 x 3 = £450

23% of £15000 = £3000 + £450 = £3450

**Finding VAT (without a calculator)**

Value Added Tax (VAT) = 20%
To find VAT, firstly find 10% and then multiply that answer by 2.

**Example** Calculate the total price of a computer which costs £650 excluding VAT

10% of £650 = £65 (divide by 10)
20% of £650 = £130 (multiply previous answer by 2)

so 20% of £650 = £130

Total price = £650 + £130 = £780

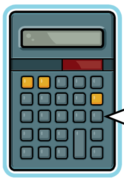
# Percentages 4

## Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

**Example 1** Find 23% of £15000

23% = 0.23 so 23% of £15000 = 0.23 x £15000 = £3450



We do **not** use the % button on calculators. The methods taught in the mathematics department are all based on converting percentages to decimals.

**Example 2** House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

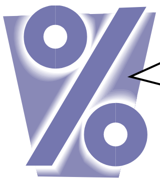
19% = 0.19 so Increase = 0.19 x £236000
= £44840

Value at end of year = original value + increase
= £236000 + £44840
= £280840

The new value of the house is £280840

# Percentages 5

## Finding the percentage



To find a percentage of a total, first make a fraction, then convert to a decimal by dividing the top by the bottom. This can then be expressed as a percentage.

**Example 1** There are 30 pupils in Class 3A3. 18 are girls.
What percentage of Class 3A3 are girls?

$$\frac{18}{30} = 18 \div 30 = 0.6 \times 100 = 60\%$$

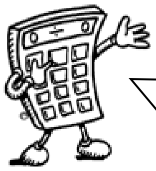
60% of 3A3 are girls

**Example 2** James scored 36 out of 44 his biology test. What is his percentage mark?

$$\text{Score} = \frac{36}{44} = 36 \div 44 = 0.81818\ldots$$

$$= 81.818..\% = 82\% \text{ (rounded)}$$

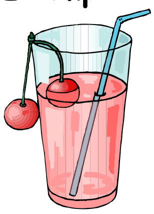
# Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

## Writing Ratios

**Example 1**



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.
The ratio of water to cordial is 4:1
(said "4 to 1")
The ratio of cordial to water is 1:4.

**Order is important when writing ratios.**

**Example 2**

In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

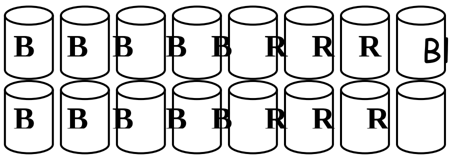
## Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

**Example 1**

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.




Blue : Red = 10 : 6
= 5 : 3

To simplify a ratio, divide each figure in the ratio by a common factor.

# Ratio 2

## Simplifying Ratios (continued)

**Example 2**
Simplify each ratio:

(a) 4:6 (b) 24:36 (c) 6:3:12

(a) 4:6
= 2:3

Divide each figure by 2

(b) 24:36
= 2:3

Divide each figure by 12

(c) 6:3:12
= 2:1:4

Divide each figure by 3

**Example 3**
Concrete is made by mixing 20 kg of sand with 4 kg cement. Write the ratio of sand : cement in its simplest form

Sand : Cement = 20 : 4
= 5 : 1

## Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

| | Fruit | Nuts | |
|---|---|---|---|
| x5 | 3 | 2 | x5 |
| | 15 | **10** | |

So the chocolate bar will contain 10g of nuts.

# Information Handling : Tables

It is sometimes useful to display information in graphs, charts or tables.

**Example 1** The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

| | J | F | M | A | M | J | J | A | S | O | N | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Barcelona | 13 | 14 | 15 | 17 | 20 | **24** | 27 | 27 | 25 | 21 | 16 | 14 |
| Edinburgh | 6 | 6 | 8 | 11 | 14 | 17 | 18 | 18 | 16 | 13 | 8 | 6 |

The average temperature in June in Barcelona is 24°C

**Frequency Tables** are used to present information. Often data is grouped in intervals.

**Example 2** Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

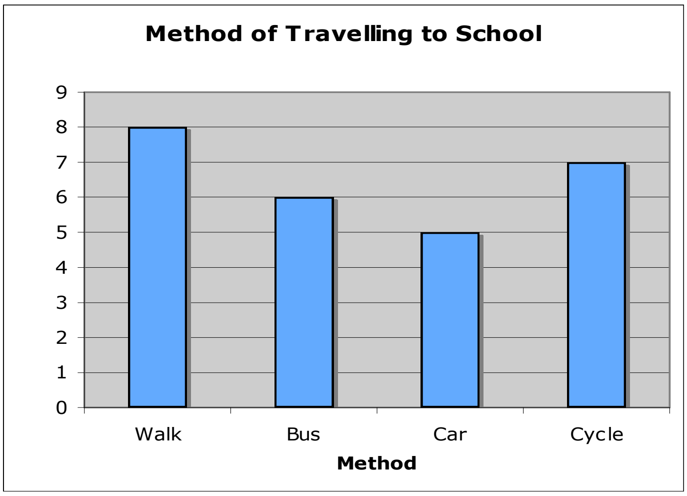
| Mark | Tally | Frequency |
|---|---|---|
| 16 - 20 | II | 2 |
| 21 - 25 | $\cancel{IIII}$ II | 7 |
| 26 - 30 | $\cancel{IIII}$ IIII | 9 |
| 31 - 35 | $\cancel{IIII}$ | 5 |
| 36 - 40 | III | 3 |
| 41 - 45 | II | 2 |
| 46 - 50 | II | 2 |

Each mark is recorded in the table by a tally mark.
Tally marks are grouped in 5's to make them easier to read and count.

Bar graphs and Histograms are often used to display data. They must not be confused as being the same.
Bar graphs are used to present discrete* or non numerical data* whereas histograms are used to present continuous data*.
See key words (Page 29) for explanation of these terms
All graphs should have a title, and each axis must be labelled.

**Example 1 Example of a Bar Graph**

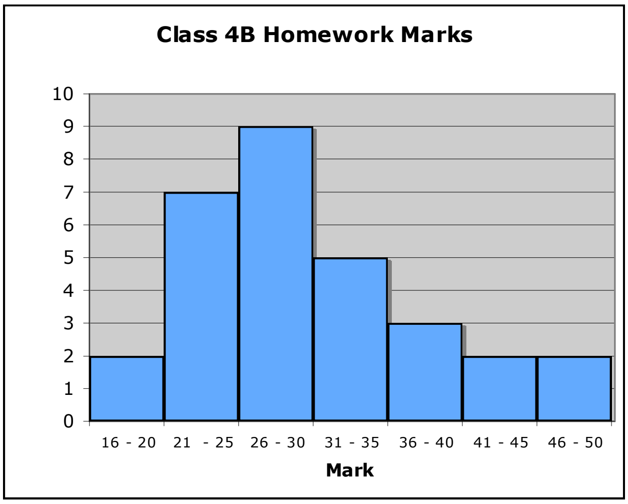
How do pupils travel to school?




An even space should be between each bar and each bar should be of an equal width. (also leave a space between vertical axis and the first bar.)

**Example 2 Example of a histogram**

The graph below shows the homework marks for Class 4B.

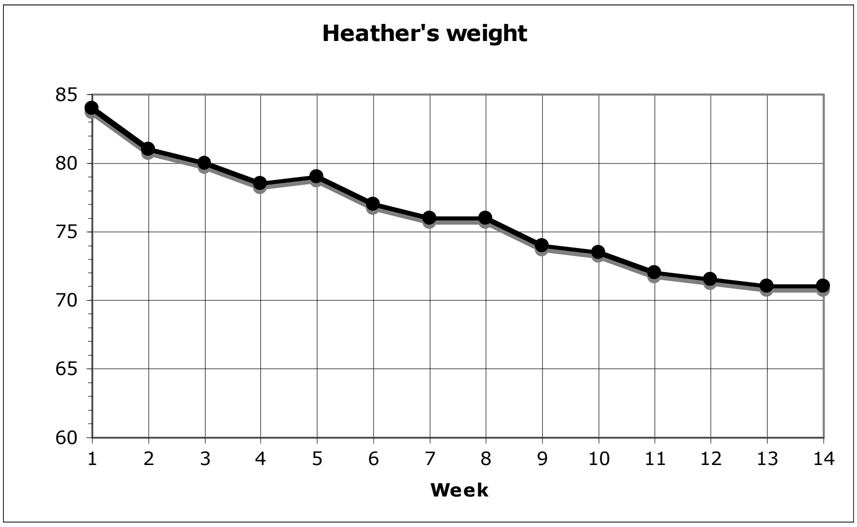



Important – there should be no space between each bar

# Information Handling : Line Graphs

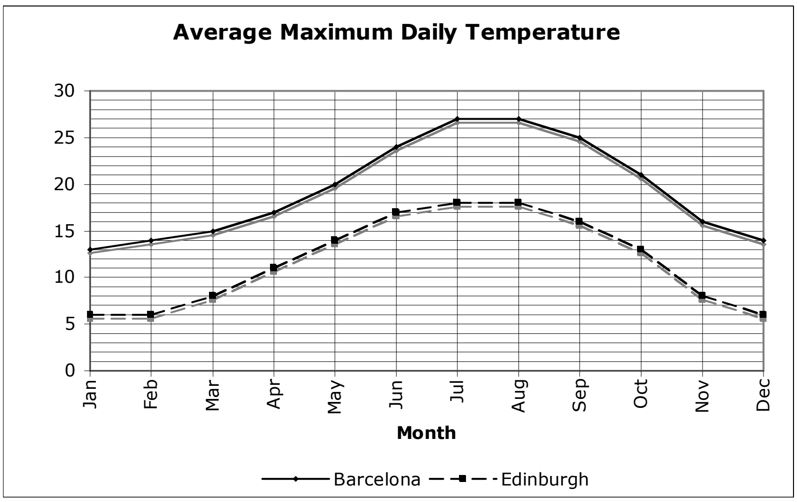
Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled. The trend of a graph is a general description of it.

**Example 1** The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.

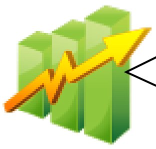



The trend of the graph is that her weight is decreasing.

**Example 2** Graph of temperatures in Edinburgh and Barcelona.



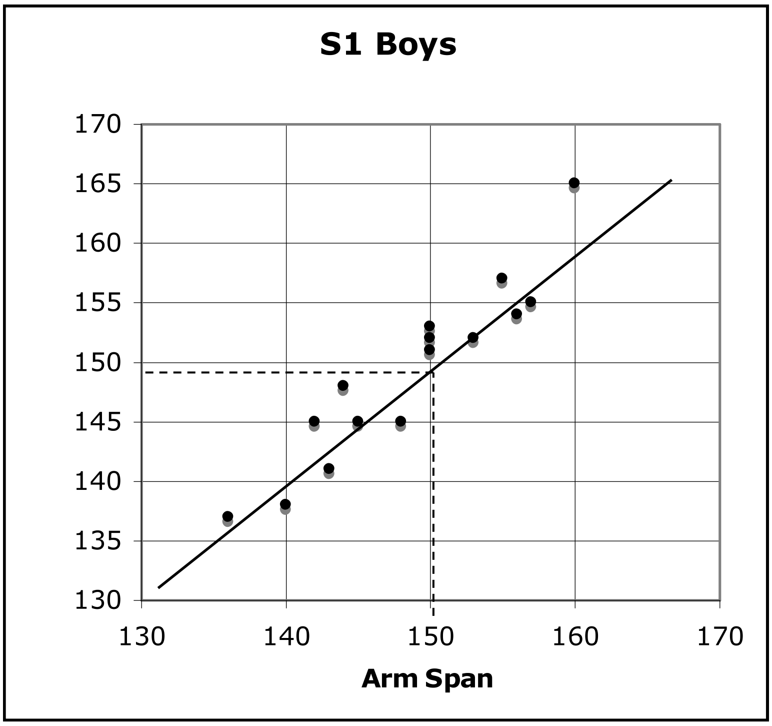
# Information Handling : Scatter Graphs



A scatter diagram is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a **correlation**.

**Example** The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph below.

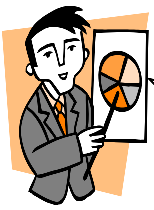
| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Arm Span (cm) | 150 | 157 | 155 | 142 | 153 | 143 | 140 | 145 | 144 | 150 | 148 | 160 | 150 | 156 | 136 |
| Height (cm) | 153 | 155 | 157 | 145 | 152 | 141 | 138 | 145 | 148 | 151 | 145 | 165 | 152 | 154 | 137 |




The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a positive correlation.

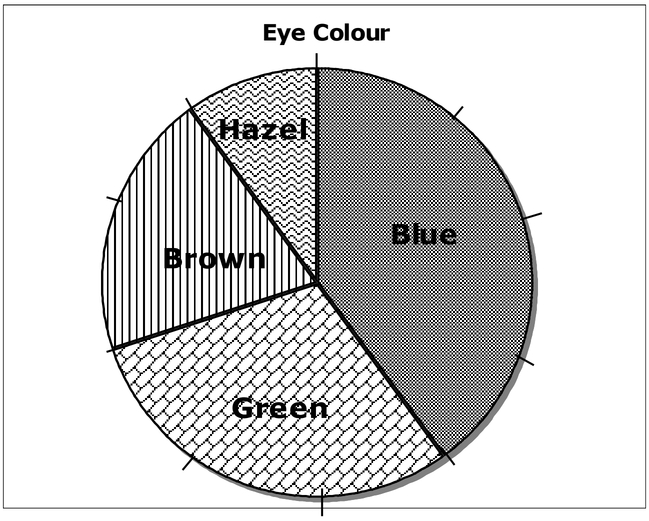
The line drawn is called the line of best fit. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm.

Note that in some subjects, it is a requirement that the axes start from zero.



A pie chart can be used to display information. Each sector (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

**Example** 30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.




How many pupils had brown eyes?

The pie chart is divided up into ten parts, so pupils with brown eyes represent $\frac{2}{10}$ of the total.

$\frac{2}{10}$ of 30 = 6 so 6 pupils had brown eyes.

If no divisions are marked, we can work out the fraction by measuring the angle of each sector.

The angle in the brown sector is 72°.
so the number of pupils with brown eyes
$= \frac{72}{360} \times 30 = 6$ pupils.
If finding all of the values, you can check your answers - the total should be 30 pupils.

# Information Handling : Averages

To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value – the mean, the median and the mode.

## Mean
The mean is found by adding all the data together and dividing by the number of values.

## Median
The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

## Mode
The mode is the value that occurs most often.

## Range
The range of a set of data is a measure of spread.
Range = Highest value – Lowest value

**Example** Class 1A scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

6, 9, 7, 5, 6, 6, 10, 9, 8, 4, 8, 5, 7

Mean = $\dfrac{6+9+7+5+6+6+10+9+8+4+8+5+7}{13}$

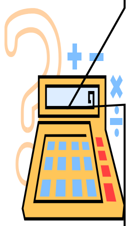
= $\dfrac{90}{13} = 6.923\ldots$ Mean = 6.9 to 1 decimal place

Ordered values: 4, 5, 5, 6, 6, 6, $\boxed{7}$ 7, 8, 8, 9, 9, 10
Median = 7

6 is the most frequent mark, so Mode = 6

Range = 10 – 4 = 6

# Scientific Notation or Standard Form



In engineering and scientific calculations you often deal with very small or very large numbers, for example 0.00000345 and 870,000,000.
To avoid writing these very long numbers a system has been developed, known as **scientific notation (standard form)** which enables us to write numbers much more concisely.

The rules when writing a number in standard form is that first you write down a number between 1 and 10, then you write × 10 (to the power of a number).

**Example**

Write 81 900 000 000 000 in standard form:

81 900 000 000 000 = $8.19 \times 10^{13}$

*It's $10^{13}$ because the decimal point has been moved 13 places to the left to get the number to be 8.19*

**Example**

Write 0.000 001 2 in standard form:

0.000 001 2 = $1.2 \times 10^{-6}$

It's $10^{-6}$ because the decimal point has been moved 6 places to the right to get the number to be 1.2

On a calculator, you usually enter a number in standard form as follows:
Type in the first number (the one between 1 and 10). Press EXP . Type in the power to which the 10 is risen.

**Interesting facts**

**Mass of Earth** = **5974200000000000000000000 kg**
= **$5.9742 \times 10^{24}$ kg**

**Mass of an electron** =**0.000000000000000000000000000000092**
= **$9.2 \times 10^{-31}$ kg**

**Mathematical literacy (Key words):** 

| | |
|---|---|
| Add; Addition (+) | To combine 2 or more numbers to get one number (called the sum or the total)<br>Example: 12+76 = 88 |
| a.m. | (Ante meridiem) Any time in the morning (between midnight and 12 noon). |
| Approximate | An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place. |
| Calculate | Find the answer to a problem. It doesn't mean that you must use a calculator! |
| Continuous Data | Has an infinite number of possible values within a selected range e.g. temperature, height, length. |
| Data | A collection of information (may include facts, numbers or measurements). |
| Discrete | Can only have a finite or limited number of possible values. Shoe sizes are an example of discrete data because sizes 6 and 7 mean something, but size 6.3 for example does not. |
| Denominator | The bottom number in a fraction (the number of parts into which the whole is split). |
| Difference (-) | The amount between two numbers (subtraction).<br>Example: The difference between 50 and 36 is 14<br>50 – 36 = 14 |
| Division (÷) | Sharing a number into equal parts.<br>24 ÷ 6 = 4 |
| Double | Multiply by 2. |

| | |
|---|---|
| Equals (=) | Makes or has the same amount as. |
| Equivalent fractions | Fractions which have the same value.<br>Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions. |
| Estimate | To make an approximate or rough answer, often by rounding. |
| Evaluate | To work out the answer. |
| Even | A number that is divisible by 2.<br>Even numbers end with 0, 2, 4, 6 or 8. |
| Factor | A number which divides exactly into another number, leaving no remainder.<br>Example: The factors of 15 are 1, 3, 5 and 15. |
| Frequency | How often something happens. In a set of data, the number of times a number or category occurs. |
| Greater than (>) | Is bigger or more than.<br>Example: 10 is greater than 6.<br>10 > 6 |
| Least | The lowest number in a group (minimum). |
| Less than (<) | Is smaller or lower than.<br>Example: 15 is less than 21. 15 < 21. |
| Maximum | The largest or highest number in a group. |
| Mean | The arithmetic average of a set of numbers - see p27. |
| Median | Another type of average - the middle number of an ordered set of data - see p27 |
| Minimum | The smallest or lowest number in a group. |
| Minus (-) | To subtract. |
| Mode | Another type of average – the most frequent number or category (see p27). |
| Most | The largest or highest number in a group (maximum). |
| Multiple | A number which can be divided by a particular number, |

| | |
|---|---|
| | leaving no remainder.<br>Example  Some of the multiples of 4 are 8, 16, 48, 72 |
| Multiply (x) | To combine an amount a particular number of times.<br>Example   6 x 4 = 24. |
| Negative Number | A number less than zero.  Shown by a minus sign.<br>Example  -5 is a negative number. |
| Numerator | The top number in a fraction. |
| Non-Numerical data | Data which is non-numerical e.g. favourite football team, favourite sweet etc. |
| Odd Number | A number which is not divisible by 2.<br>Odd numbers end in 1, 3, 5, 7 or 9. |
| Operations | The four basic operations are addition, subtraction, multiplication and division. |
| Order of operations | The order in which operations should be done.  BODMAS (see page 8). |
| Place value | The value of a digit dependent on its place in the number.<br>Example: in the number 1573.4, the 5 has a place value of 100. |
| p.m. | (Post meridiem) Any time in the afternoon or evening (between 12 noon and midnight). |
| Prime Number | A number that has exactly 2 factors (can only be divided by itself and 1).  Note that 1 is not a prime number as it only has 1 factor. |
| Product | The answer when two numbers are multiplied together.<br>Example: The product of 5 and 4 is 20. |
| Remainder | The amount left over when dividing a number. |
| Share | To divide into equal groups. |
| Sum | The total of a group of numbers (found by adding). |

| | |
|---|---|
| Total | The sum of a group of numbers (found by adding). |

# TIMELINE
## The following topics are covered in S1 and S2 in the Mathematics department:

| Topic | Month |
|---|---|
| **Decimals**<br>• Looking back<br>• Addition & subtraction without a calculator<br>• Rounding to one decimal point<br>• Multiplying & dividing by 10, 100<br>• Multiplying & dividing decimals by whole numbers<br>• Multiplying &dividing on a calculator | August |
| **Angles**<br>• Looking back<br>• Corresponding angles<br>• Alternate angles | September |
| **Letters & Numbers**<br>• Order of operations (BODMAS)<br>• Looking back<br>• Like terms | September |
| **Position & Movement**<br>• Looking back<br>• Bearings<br>• Coordinates in four quadrants | September - October |
| **Whole numbers**<br>• Whole number calculations<br>• Squares, cubes and square roots<br>• Factors & prime factors<br>• Working with integers: below zero<br>• Subtracting integers | October - November |

| **Topic** | **Month** |
|---|---|
| • Multiplying integers<br>• Dividing integers<br>• Sum, difference, product, quotient | |
| **Tiling and Symmetry**<br>• Looking back (line symmetry)<br>• Rotation & rotational symmetry<br>• Translation & translational symmetry<br>• Enlargement & reduction | November |
| **Decimals**<br>• Adding & subtracting<br>• Rounding to more than one decimal place<br>• Further multiplication & division<br>• Significant figures | November |
| **Letters & Numbers**<br>• Repeated adding & multiplying<br>• Evaluating squares<br>• Removing brackets<br>• Factorising expressions | December |
| **Measuring length**<br>• Looking back<br>• Reading scales<br>• Calculating perimeter using scales | December |
| **Information Handling**<br>• Looking back<br>• Organising information<br>• Summarising/comparing data: mean<br>• Summarising/comparing data: median, mode & range<br>• Grouped frequency tables | January |
| **Time & Temperature**<br>• Looking back<br>• Timetables<br>• Using stopwatches | January - February |

| Topic | Month |
|---|---|
| • Calculating speed<br>• Calculating distance<br>• Calculating time<br>• Which formula? | |
| **Area**<br>• Looking back<br>• Using formulae<br>• Right-angled triangles<br>• Any triangle | February |
| **Solving Equations**<br>• Looking back<br>• Keeping your balance<br>• Inequations<br>• Solving more equations<br>• Negative numbers & expressions<br>• Negative numbers & equations<br>• Brackets & equations | February - March |
| **The Triangle**<br>• Looking back<br>• The sum of angles in a triangle<br>• Drawing triangles<br>• Measuring heights & distance<br>• The exterior angles of a triangle | March |
| **Three Dimensions**<br>• Looking back<br>• Pyramids & prisms<br>• Drawing solids<br>• Skeletal models<br>• Nets of pyramids & prisms<br>• Surface area<br>• Volumes of cubes & cuboids | April |
| **Information Handling 2**<br>• Looking back<br>• Displaying data: line graphs | May |

| Topic | Month |
| --- | --- |

- Displaying data: pie charts

**Fractions & Percentages** — May

- Looking back
- Finding the fraction of an amount
- Calculating the percentage of an amount
- Using percentages
- Rational numbers
- Equivalent fractions & mixed numbers
- Adding & subtracting common fractions

**Letters, Numbers & Sequences** — May - June

- Looking back
- The shape of numbers
- The nth term from multiples
- Rule-making
- Problem-solving

**Two Dimensions** — June

- Looking back
- Square & rectangle
- Rhombus & kite
- Parallelogram & trapezium

**Ratio & Proportion** — August

- Looking back
- Ratios
- Unitary ratios
- Sharing
- Direct proportion
- Graphing direct proportion
- Inverse proportion
- Mixed examples